Some issues in the ghost condensation scenario

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Plan.

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I. Why to modify gravity?

Recent satellite experiments revealed that on the large distances gravity behaves in a rather bizarre way:

- Dimming supernova and acceleration of the Universe

- Rotational curves

- Pioneer anomalous acceleration (???)

Usual explanations: new forms of energy (Dark Energy) and matter (Dark Matter)

But the situation is not new!

- in 1800's observed precession of Mercury perihelion

- First explanation $->$ Dark Planet: Vulcan

- The right answer was not the Dark Planet, but the modification of Newtonian gravity: Einstein's theory of GR

- Is it possible to modify Einstein's gravity in the infrared in a theoretically and experimentally viable way to address these issues? (especially C.C.)

II. Different proposals.

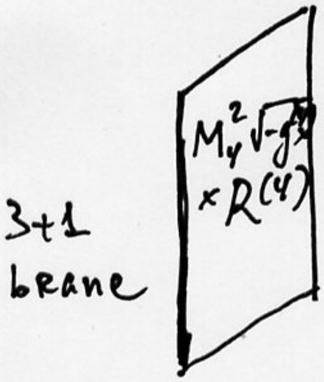
- Pauli-Fiertz mass term to the Einstein's gravity:

$$S = \int d^4x \sqrt{-g} M_{pl}^2 R + F^4 \int d^4x (h_{\mu\nu}^2 - h^2) \, .$$

Graviton gets the mass

$$m_g^2 \sim \frac{F^4}{M_{pl}^2}$$

- Dvali-Gabadadze-Porrati (DGP) model



$\longrightarrow$ $M_5^3 \sqrt{-G}\, R^{(5)}$

infinite volume 5'th dim

$$V(r) \sim \begin{matrix} 4D & \text{for } r < r_c \\ 5D & \text{for } r > r_c \end{matrix} \qquad r_c \sim \frac{M_4^2}{M_5^3}$$

However, in both theories matter is coupled with gravitational strength to a new scalar d.o.f., which becomes strongly coupled at an intermediate scale:

$$\Lambda^{-1} = \begin{pmatrix} m_g^2 M_{pl} \\ r_c^{-2} M_{pl} \end{pmatrix}^{1/3} \sim 1000 \text{ km}$$

=> breakdown of the effective field theory at larger distances

Other possibilities:

- $R -> f(R, R_{\mu\nu}R^{\mu\nu})$
- $R -> f(r, \Box R, ...)$

Either equivalent to some scalar-tensor theory or populated with ghost d.o.f.

III. Ghost condensation and modification of gravity in the infrared

There is a way to modify gravity in the infrared in a theoretically consistent way! The price to pay: spontaneous Lorentz invariance breaking

Nima Arkani-Hamed, Hsin-Chia Cheng, Markus Luty, Shinji Mukohyama, JHEP **0405**, 074 (2004); [arXive: hep-th/0312099]

IIIa. Ghost Condensation.

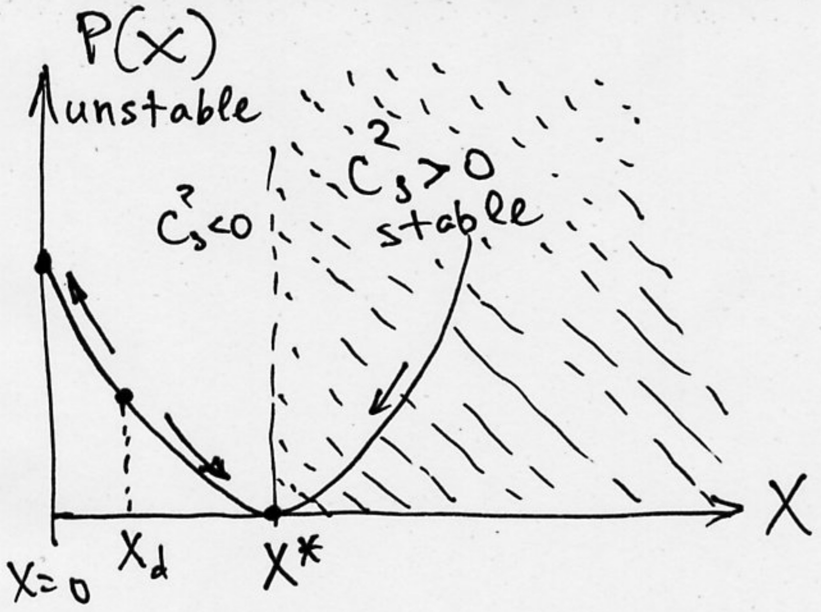
Consider first the Lagrangian $\mathcal{L} = M^4 P(X)$, where M is some scale, $X = (\partial\phi)^2/M^4$; $P(X) = -X + \mathcal{O}(X^2)$, when $X \ll 1$

This is a "ghost-like" Lagrangian; the equation of motion is

$$\partial_\mu \left(\sqrt{|g|} P'(X) \partial^\mu \phi \right) = 0$$

In the homogeneous case:

$$P'(X)\dot{\phi} = \frac{c}{a^3(t)} \to 0$$



$$\mathcal{E} = 2XP'(X) - P(X)$$
$$\mathcal{E}(X^*) = -P(X^*)$$

Consider excitations π

$$\phi = M^2 t + \pi$$

Expanding the Lagrangian around X^*:

$$\mathcal{L} = \left[P'(X^*) + 2X^* P''(X^*)\right]\dot{\pi}^2 - P'(X^*)(\nabla\pi)^2$$

The coefficient in front of ∇-term vanishes, the sign in front of a $\dot{\pi}^2$-term is positive to the left of X^*.

What about higher derivative corrections?

$$\mathcal{L} = \sqrt{|g|}(P(X) + Q(X)R(\Box\phi))$$

The equation of motion is

$$\partial_t \left(a^3 \left[(P'(\dot\phi^2) + Q'(\dot\phi^2)R(\ddot\phi + 3H\dot\phi))2\dot\phi - \partial_t(Q(\dot\phi^2)R'(\ddot\phi + 3H\dot\phi)\right]\right) = 0 \,,$$

which splits as $a \to \infty$ into

$$\left[P'(\dot\phi^2) + Q'(\dot\phi^2)R(\ddot\phi + 3H\dot\phi)\right] 2\dot\phi \to 0$$

and

$$Q(\dot\phi^2)R'(\ddot\phi + 3H\dot\phi) \to \text{const}$$

It is not hard to verify that the first part is a coefficient in front of ∇-term: it vanishes again.

Some caution: this coefficient oscillates with the frequency $\sim M$:

$$\sim \frac{1}{Mt}\sin(Mt)$$

Not a big problem, explanation later.

In the $(\Box\phi)^2$ there are terms $(\nabla^2\pi)^2$ which will dominate over vanishing $P'(X)(\nabla\pi)^2$ term.

The Lagrangian for π near X^* at quadratic level:

$$\mathcal{L} = \frac{\dot{\pi}^2}{2} - \frac{\lambda^2(\nabla^2\pi)^2}{M^2} + \text{higher in } \pi's$$

Thus, unusual dispersion relation:

$$\omega^2 \sim \frac{k^4}{M^2}$$

At this point we have

- Ghost condensate $X = X^*$; X^* is the time-like, =>

- Preferred reference frame where $M^4X^* = \dot{\phi}^2$; this is the same as cosmological/CMBR reference frame

- Spontaneous breaking of the Lorentz invariance

IIIb. Gravity modification.

Condensate spontaneously breaks Lorentz invariance;

When π's are mixed with gravity there is an additional gravitational scalar d.o.f.:
graviton= 2 tensor d.o.f. +1 scalar d.o.f.

The Lagrangian is

$$\mathcal{L} = -\frac{(\nabla\Phi)^2}{2} + \frac{1}{2}(m\Phi - \dot{\pi})^2 - \frac{\lambda^2(\nabla^2\pi)^2}{2M^2} + ...,$$

where $\Phi = h_{00}/2$ and $m = M^2/\sqrt{2}M_{pl}$.

$$< \Phi\Phi >= -\frac{1}{\vec{k}^2} \times \left(1 - \frac{\lambda^2 m^2 \vec{k}^2}{M^2\omega^2 - \lambda^2\vec{k}^4 + \lambda^2 m^2 \vec{k}^2}\right)$$

The second part leads to modification of the Newton potential. The dispersion relation is modified!:

$$\omega^2 = -\frac{\lambda^2 m^2}{M^2}\vec{k}^2 + \frac{\lambda^2\vec{k}^4}{M^2}$$

There is a tiny instability band! $(\omega_J \sim M^3/M_{pl}^2)$

IV. Symmetry arguments.

The construction in the previous section can be obtained from the symmetry principles:

- Break time reparametrization invariance, leave only spatial diffeomorphisms: $\vec{x} \to \vec{x}'(t, \vec{x})$

- Write all terms consistent with this residual symmetry; at leading order:

$$h_{00}^2, \quad h_{ij}^2$$

- Next to leading (using ADM $3+1$ split):

$$K_{ii}^2, \quad K_{ij}^2, \quad K_{ij} \to \frac{1}{2}(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j} - \partial_i \partial_j \pi),$$

which have $(\nabla^2 \pi)^2$.

- Term h_{0j}^2 which contain $(\nabla \pi)^2$ is not invariant, => no term $(\nabla \pi)^2$ in the action

V. The effective field theory.

- The ghost condensate itself ($P(X)$) does not lead to gravity modification; only adding higher derivative terms (like $(\nabla^2\pi)^2$) one modifies gravity

- Is the effective field theory sketched before a well behaving in the infrared?

- First note that due to unusual dispersion relation some quantities are scaled differently with the energy:

$$E \to sE, \ t \to s^{-1}t, \ x \to s^{-1/2}x, \ \pi \to s^{1/4}\pi$$

- Working out the Lagrangian for π up next to quadratic level one finds that the most dangerous operator is $\dot{\pi}(\nabla\pi)^2$; it scales as $s^{1/4}$, => (barely) irrelevant

The rest of operators are even more irrelevant, therefore the expansion is under control.

Two more words of caution, if one formulates the theory in a covariant way, i.e. with the $(\Box\phi)^{2n}$'s

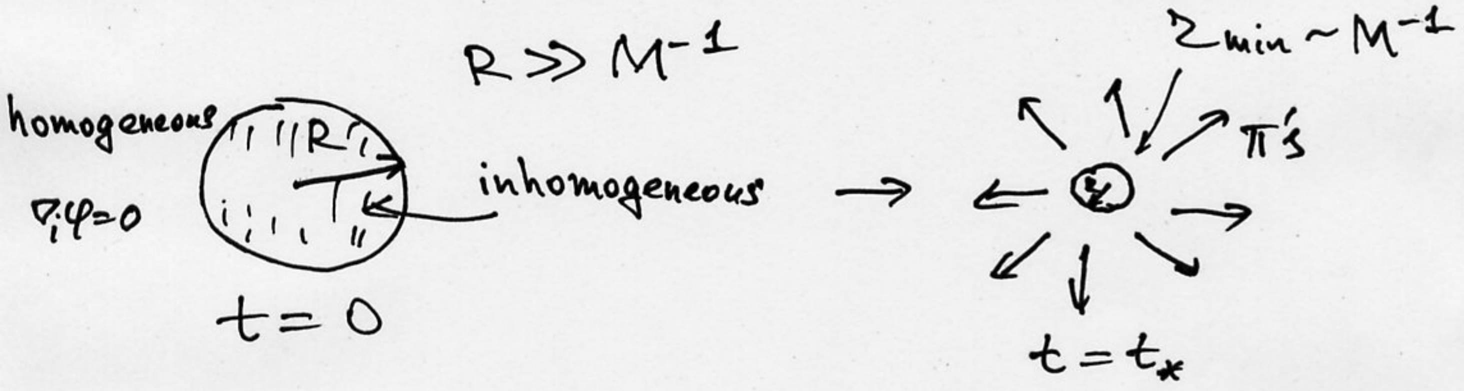
- The theory truncated at some finite set of operators will lead to the equation of motion for the background, which in general are oscillatory and $\omega \sim M$; this does not mean instability but rather that one can't consistently decide whether it is or isn't there from the low-energy theory alone

- There is a "strong coupling" regime when $H/M > 1$; this comes out of a $(\Box\phi) = \ddot{\phi} + 3H\dot{\phi} \to 3HM^2$; contribution to the action from 'homogeneous' part gets larger than 'spatial' part; this effective field theory, for example, may not exist in De Sitter with large H

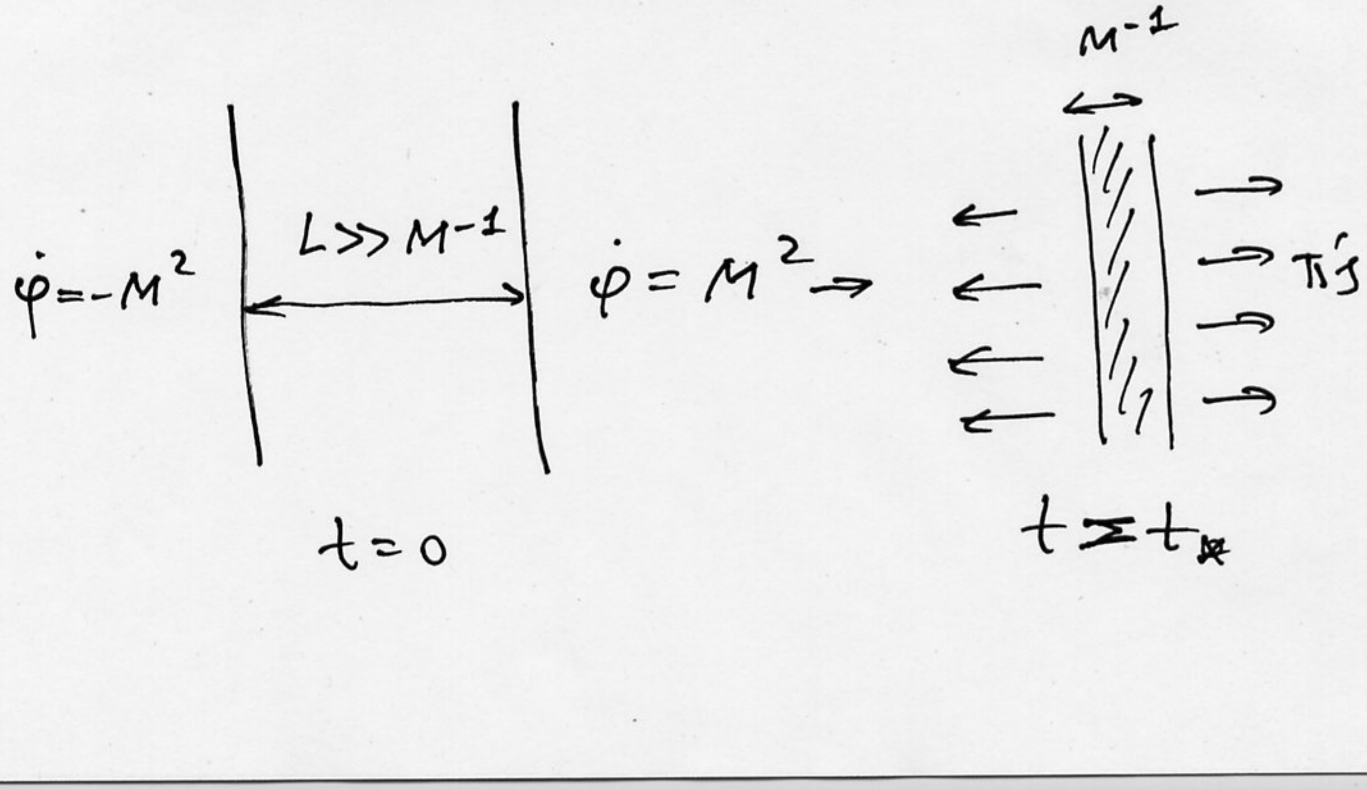
VI. Holes/domain walls in the ghost condensate.

There is an interesting physics in two aspects:

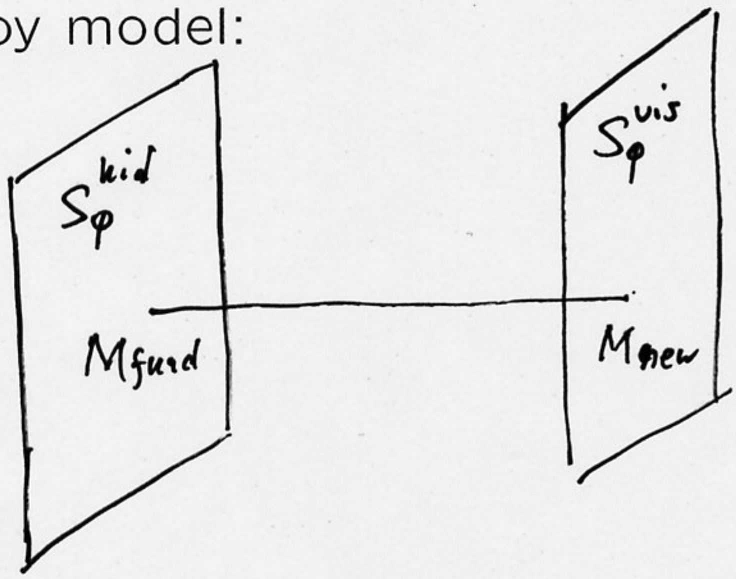
- Evolution of the "bubbles" (Rubakov et.al)



- There are two vacua with the same energy density: $\dot{\phi} = \pm M^2, =>$ domain walls:



A toy model:



$$\sqrt{-g^{vis}}\, g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \to e^{-2kr_c\pi}\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

$$\varphi \to e^{kr_c\pi}\varphi$$

$$S_\phi^{hid} = \sqrt{-g^{hid}}[-\frac{(\partial\phi)^2}{2} + \frac{(\partial\phi)^4}{4M_{fund}^4} - \frac{(\lambda\Box\phi)^2}{M_{fund}^2}]$$

and

$$S_\phi^{vis} = \sqrt{-g^{vis}}[-\frac{(\partial\phi)^2}{2} + \frac{(\partial\phi)^4}{4M_{fund}^4} - \frac{(\lambda\Box\phi)^2}{M_{fund}^2}],$$

$$g_{\mu\nu}^{vis} = e^{-2k\pi r_c}\eta_{\mu\nu}$$

Canonically normalizing regular kinetic term on the visible brane, one obtains the relation

$$M_{new} = M_{fund}\exp(-kr_c\pi)$$

Note, that the same is true for an arbitrary case with

$$\mathcal{L} = \sqrt{|g|}(P(X) + Q(X)R(\Box\phi))$$

VII. The scale M.

Two cases:

- $\rho_{cond} \neq 0$, the equation of state is $p = -\rho$, => $M \sim 10^{-3}$eV: k-essence, which drives current acceleration

- M could be much larger if the energy of the condensate at X^* vanishes

What are constraints in the second case?

Requirement that $H_0 > \omega_J$ leads to: $M \leq 10$ MeV- but not a stringent constraint; still M is much smaller than fundamental scales, e.g. M_{pl}

VI. Some open problems.

- A viable UV completion
- Is π a good candidate for the dark matter?
- Is there a tight bound on the low energy effective scale M?
- Quantum stability, holes in the ghost condensate/domain walls.
- Accretion of the ghost condensate/π matter on the black hole